Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Problem Solving -I

Subject Code : 5SC02MTE2 **Branch: M.Sc.(Mathematics)**

Semester : 2 **Date : 17/05/2016** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **(07)**

a. What is a rank of $n \times n$ identity matrix? **(01)**
b. State Cayley-Hamilton Theorem. **(01)**
c. Give relation between trace of matrix and Eigen value of matrix. **(01)**
d. Find the sum of the series $\sum_{k=1}^{\infty} \frac{1}{k!}$. **(01)**
e. Define Hermitian matrix. **(01)**
f. Find the solution of $\frac{dy}{dx} = xy + 1$. **(02)**

Q-2 **Attempt all questions** **(14)**

a) Let V be the vector space of polynomials over **R** of degree less than or equal to n, for $p(x) = a_0 + a_1x + \cdots\cdots + a_nx^n$ in V, defined a linear transformation T:V→V by $T(p(x)) = a_0 - a_1x + \cdots\cdots + (-1)^n a_n x^n$. Then show that T is 1-1, onto and invertible. **(06)**

b) Evaluate: $\int_{\Gamma} \frac{z^3+2z}{(z-a)^3} dz$. **(05)**

1) When $a \in \Gamma$.
2) When a lies out side Γ.

c) Let $p(x) = x^4 - 4x^3 + 2x^2 + ax + b$ .Suppose that for all root β of $p(x)$, $1/\beta$ is also a root of $p(x)$. Find the values of a and b. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a) Let J denote a 101×101 matrix with all the entries equal to 1 and let I denote the identity matrix of order 101. Then find the determinant of $J - I$. **(06)**

b) Let $u(x+iy) = x^3 - 3xy^2 + 2x$ for which find v such that $u + iv$ is holomorphic function on **C.** **(05)**

c) Let K be the positive integer then find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(n!)^K}{(Kn!)}$. **(03)**

Q-3 **Attempt all questions** **(14)**

a) Expansion of $f(x) = \frac{1}{(z-1)(z-2)}$ in the regions **(07)**

i) $|z| < 1$

ii)$1 < |z| < 2$

b) Evaluate : $\int_C \frac{z-3}{z^2+2z+5} dz$ **(07)**

(1) $C: |z + 1 - i| = 2$

(2) $C: |z + 1 + i| = 2$

OR

Q-3 **Attempt all Questions** **(14)**

a) Solve :$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ **(07)**

b) Evaluate: $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta$. **(07)**

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a. Let $(z) = \frac{5z-2}{z(z-1)}$. Find residue of f at each point. **(02)**

b. Find the determinant of $n \times n$ permutation matrix $= \begin{bmatrix} & & & & 1 \\ & & & 1 & \\ & & 1 & & \\ & \cdots & & & \\ 1 & & & & \end{bmatrix}$. **(01)**

c. Solve $(D^2 - 6D + 9)y = 0$. **(02)**

d. Given that the matrix $A = \begin{bmatrix} \alpha & 1 \\ 2 & 3 \end{bmatrix}$ has 1 as eigen value . compute its trace and determinant. **(02)**

Q-5 **Attempt all questions** **(14)**

a) Find the Residue of $z \cos\frac{1}{z}$ at $z = 0$. **(05)**

b) Let A be a 3×3 upper triangular matrix whose diagonal entries are 1,2 and 3.express A^{-1}as in terms of I, A and A^2. **(05)**

c) Let $p(z) = a_0 + a_1 z + \cdots\cdots + a_n z^n$ and $q(z) = b_1 z + \cdots\cdots + b_n z^n$ be complex polynomials. If a_0 , b_1 are non zero complex numbers then the residue of $\frac{p(z)}{q(z)}$ at 0 is ____________. **(04)**

OR

Q-5 **Attempt all Questions** **(14)**

a) Solve $(D^2 - 5D + 6)y = e^x \cos 2x$. **(05)**

b) Find $\int_{|z+1|=2} \frac{z^2}{4-z^2} dz$. **(05)**

c) Show that the polynomial of odd degree with real coefficient must have atleast one real root. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Solve :$(2x^2 + 3y^2 - 7)xdx - (3x^2 + 2y^2 - 8)ydy = 0$. **(05)**

b) Evaluate power series of $f(z) = \frac{1+2z}{z^2+z^3}$ around $0 < |z| < 1$. **(05)**

c) Let $A = (a_{ij})$ be 2×2 lower triangular matrix with diagonal entries $a_{11} = 1$, $a_{22} = 3$. If $A^{-1} = (b_{ij})$ then find the values of b_{11} and b_{22}. **(04)**

OR

Q-6 **Attempt all Questions** **(14)**

a) Evaluate :$\int_C \frac{\sin^2 z}{(z-\pi/6)^3} dz$,where C is circle $|z| = 1$. **(05)**

b) Solve : $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$ **(05)**

c) Let $z = x + iy \in \boldsymbol{C}$ and let f be defined by $f(z) = y - x - 3x^2 i$. If C is a straight line joining $z = 0$ to $z = 1 + i$. Compute $\int f(z) dz$. **(04)**